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DISCUSSIONS.

In a recent number of the MONTHLY, Professor E. J. Moulton considered the nature of a course in advanced calculus which should serve as a bridge between the work in elementary calculus and more advanced subjects. In the first discussion below, Professor Gillespie presents the claims of elementary differential equations for adequately effecting this transition. The content which he proposes is by no means that of the course in elementary differential equations as it is usually given; neither does it coincide with the usual course in advanced calculus. Further expressions of opinion on the questions raised will be welcome.

Among the topics presented in courses common to the secondary and collegiate fields, probably no other is so difficult, and at times so discouraging, to teach satisfactorily, as mathematical induction. Professor Weaver, in the second discussion of this number, gives an amusing account of an analogy from life, which may be useful in illustrating the essential nature of the method of proof involved.

In the third discussion Professor Noble considers the process of altering the strength of a mixture by continuous addition of quantities of one component. Mathematically the situation turns out to involve a neat illustration of the notion of a derivative, leading to a simple differential equation; some of our readers will find here a useful supplementary problem for the class-room.

I. ADVANCED CALCULUS OR DIFFERENTIAL EQUATIONS.

By D. C. GILLESPIE, Cornell University.

The second course in calculus as it is often given,—following Goursat-Hedrick, *Mathematical Analysis*, Vol. I, let us say to be explicit,—breaks suddenly with what has gone before. It does not appeal to everyone as a natural continuation of previous work. The point of view is different, the emphasis is different, and even among those who have done well in the more elementary courses there are students with whom it is not successful. Professor E. J. Moulton's suggestive paper in the December number of the MONTHLY proposes a plan which would accomplish much that it is desirable to accomplish by the second course in the calculus and at the same time make the change less abrupt.

Even in the case of the future mathematicians, Professor Moulton feels, "It is too early in the student's career to introduce the critical attitude of higher mathematics." I am not sure that this phrase is quite clear. The student is perhaps fortunate to escape altogether the attitude that is more concerned with detecting error than with acquiring knowledge. On the other hand the desire and ability to analyze and see on what assumptions theorems are founded and how the results follow from the assumptions are, at every stage of the student's development, great assets. The point is, it seems to me, that while the teacher may encourage this desire he can not introduce this ability. The scales are not going to fall from the student's eyes at the teacher's command; they wear off, and indeed slowly. The plan proposed has rather this clear advantage, that, instead of halting the progress of the student until he has mastered the fundamental concepts of the calculus, it offers him a course useful and interesting in itself where the ideas of the calculus are continually applied.

Somewhat the same purpose may be accomplished, I think, by an elementary course in differential equations. At most collegiate institutions such a course is given each year. This course may quite naturally cover a good deal of the outline proposed for the second course in calculus.

It is true in my experience with both text books and lectures that courses in differential equations divide themselves rather sharply into two classes: the first is purely formal, where the time is spent in the monotonous and, if long continued, rather deadening work of learning schemes of integration; the second consists in courses devoted to the study of existence theorems and properties of solutions. One must go to books on differential geometry or mechanics or mathematical physics to find the calculus used to state problems and the geometrical and physical notions involved further employed to aid in the solution and in the interpretation of results. It would however be necessary for the purpose at hand to include these applications of calculus under the name elementary differential equations.

The course in differential equations would include methods of integration. It should perhaps include the power series existence theorem for ordinary differential equations of the first order. There could be added many problems from geometry, mechanics and physics which are not stated as differential equations but which the student himself must formulate, solve and interpret. Such a course is a continual review of the ideas of function, continuity, derivative, integral. Families of curves, their trajectories, envelopes and singularities would be included. The plane pendulum problem gives a natural introduction to elliptic integrals and functions. The brachistochrone or some similar problem takes one at least to the Euler equation in the calculus of variations. The existence theorem mentioned or an approximation method leads to some study of Taylor's series and infinite series generally.

The student should acquire in a course of this nature considerable knowledge and technique, as well as a better understanding of the calculus. This would allow more time for the essentially slow process of the accurate statements and proofs of advanced calculus when it follows.

II. Note on the Teaching of the Principle of Mathematical Induction.

By Warren Weaver, University of Wisconsin.

The writer well remembers when he first met a proof by mathematical induction. He felt as it he had been introduced to a scientific three-shell artist. While he could find no definite loop-hole in the argument it seemed too mysterious and unreal a process to have actually effected any definite proof. The attitude of classes meeting this principle for the first time seems to justify the conclusion that this is no unusual circumstance. The fact that some ridiculously simple